

Stellar footprints of a variable G ¹

Diego F. Torres²

*Departamento de Física, Universidad Nacional de La Plata,
C.C. 67, 1900 La Plata, Argentina*

Abstract

Theories with varying gravitational constant G have been studied since long time ago. Among them, the most promising candidates as alternatives of the standard General Relativity are known as scalar-tensor theories. They provide consistent descriptions of the observed universe and arise as the low energy limit of several pictures of unified interactions. Therefore, an increasing interest on the astrophysical consequences of such theories has been sparked over the last few years. In this essay we comment on two methodological approaches to study evolution of astrophysical objects within a varying- G theory, and the particular results we have obtained for boson and white dwarf stars.

¹This essay received an “honorable mention” in the 1999 Essay Competition of the Gravity Research Foundation - Ed.

²Electronic address: dtorres@venus.fisica.unlp.edu.ar

1 Introduction

The idea of a varying gravitational constant G has been in the physicist's minds since long time ago, when Dirac proposed the large number hypothesis [1]. It states that the ubiquity of some large dimensionless numbers $\sim \mathcal{O}(10^{40})$ -, arising from the combination of micro and macrophysical parameters, is not a coincidence but an output of an underlying time variation in $e^2 G^{-1} m_p$, where e is the unit charge and m_p is the proton mass. Well posed relativistic theories admitting time variation in the fundamental constants of Nature had to wait for about twenty years since those ideas, until they were introduced, in their present form, by Brans and Dicke [2]. The gravitational “*constant*” then became a variable field.

One of the first motivations to replace General Relativity (GR) for a Brans-Dicke (BD) or, more generally, for a scalar-tensor (ST) theory, was a seeming discrepancy between observations and the weak field GR predictions. Time went by, these differences vanished, and the main motivation shifted to cosmology.

In ST theories, the gravitational action has a free parameter, $\omega(\phi)$, called *coupling function*. For particular values of this parameter, these theories have cosmological solutions which are entirely compatible with all current gravitational tests: solar-system-based, gravitational lensing, strong field pulsar tests, and nucleosynthesis [3]. Still, these theories may notably deviate from GR, and might have played a crucial role in the early universe. The scalar field may be a source of inflation and, moreover, some ST theories are the low energy limit of unified pictures. In general, we can say that, albeit severely constrained, a very slow time variation of G cannot be discarded, especially when cosmological time intervals are considered. Others *constants* of Nature could well share this striking result (see, for instance, the observational study on possible space-time variation of the fine structure constant [4] and the consequent theoretical interest in a variable light speed [5]).

2 Astrophysical modelling a variable G

When G is assumed to vary on cosmological intervals of time, it is natural to expect that this will influence the evolution of all non-transient astrophysical objects. A few years ago, the possibility of the existence of *gravitational memory* was introduced through the consideration of what happens to

black holes, during the evolution of the universe, if G evolves with time [6]. One possibility is that the black hole evolves quasi-statically, in order to adjust its size to the changing G . If true, this means that there are no static black holes, even classically, during any period in which G changes. Another possibility results when the local value of G within the black hole is preserved while the asymptotic value evolves with a cosmological rate. This would mean that the black hole remembers the strength of gravity at the moment of its formation. It is immediate to extend this analysis to any object, like neutron or other kind of stars.

Then, the general problem we face is to have a complete solution, relativistic and space-time dependent, for the metric, scalars, and matter fields, able to represent the evolution of a given object through cosmic time. However, to solve this problem is far from being an easy task: in the usual applications, we assume fields that are either spatially-constant but time-varying (cosmology), or spatially-varying but time-independent (astrophysics). Any real situation would require, however, a combination of both and then, no global conservation law is in general available. Moreover, it worsens with the complexity of the internal structure of the object.

To consider these astrophysical scenarios, we must adopt some simplifications. We shall explore two approaches: either we consider simple stellar objects within a full relativistic gravitation, or reduce the complexity in the underlying theory (through Newtonian approximations), augmenting that of the object under study. This latter case might render direct observational consequences.

In the left side of Fig. 1, we illustrate the first approach: we assume a relativistic theory, say a scalar-tensor one, and study simple theoretical constructs, with hardly testable predictions. We expect that the results so obtained may be at least indicative of what happens with usual stars. If G varies, the value of the effective gravitational coupling far out from the star must not necessarily be the Newton constant. On the contrary, it must take the value given by the evolution of a cosmological model of the same gravitational theory (and at the same time) in which the object is modelled. This has the immediate consequence of changing the boundary conditions. It lets us to approximately study objects through different cosmic eras by starting from relativistic field equations and modifying the asymptotic value of G ; i.e. we are only capable to construct sequences of static configurations.³

³We cannot always state, however, that these sequences adequately represent the real evolution of the object, as

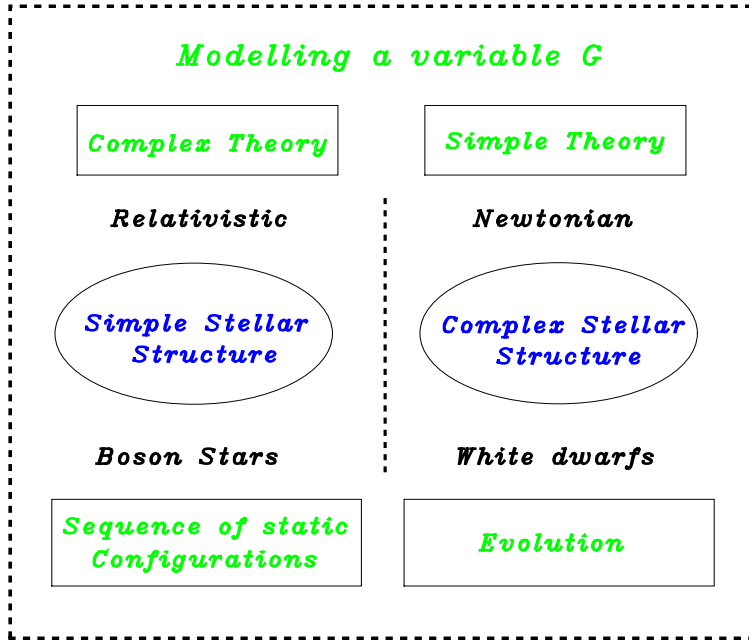


Figure 1: Two different approaches to uncover the influence of a variable G on astrophysical structures. In all real cases, G varies in space because of the existence of the massive object, and in time because the theory in which the object is modelled provides a cosmological evolution. For usual situations, these two variations are of the same order, amounting few percents of the present value of G . The last box of both paths signals which kind of approximation is involved. On the left, a relativistic treatment disregard an explicit time dependence, and thus yields to sequences of static configurations. On the right, the space dependence of G is overwritten with a cosmological time evolution, and as the treatment is Newtonian, we may compute actual evolution of all matter fields.

We have studied this case for boson stars: the analogue of a neutron star formed when a large collection of bosonic particles becomes gravitationally bound. Although such configurations were introduced in the 60's [7], the current interest was prompted by the proof that, provided the scalar field has a self-interaction, boson star masses could be of the same order of magnitude as, or even much greater than, the Chandrasekhar mass [8]. It seems possible for these stars to form through the collapse of a scalar field or gravitational cooling [9], though little is known yet. Since boson stars are easier to compute than black holes, because of the absence of singularities and horizons, we were able to numerically solve their equilibrium structure in several scalar-tensor theories, for different cosmic times. We have explicitly shown that these configurations are sensitive to small variations in the boundary condition for G , these changes amounting for a few percents of their own

we can do below when considering white dwarfs. In that case, because of appropriate approximations, the matter fields are already functions of time.

masses. Assuming the past value of the coupling constant G to be greater than the present one, we found that the mass and particle number of static configurations, at fixed central density, increase from earlier to later times. Although stable stars may exist at all times, as time goes by, models with a given central density moves towards the stable branch. Also, the radius-mass relationship is appreciably modified. Our latest results, assuming that the system evolves conserving the number of particles, show that the mass and the central density must decrease in time (considering a constant mass unit). The reduction in central density seems reasonable when we recall the force balance in polytropic stars: since gravity is reducing in strength, the equilibrium configurations can become more diffuse and hence drop in central density. Detailed discussion of these results may be found in Refs. [10]. All in all, despite we are not able to answer which phenomenon (memory or quasi-static evolution) actually happen, we confirm that these equilibrium spheres are sensitive to a very slow cosmological variation of G .

To judge whether a quasi-static evolution is a feasible scenario, we can compare the free-fall time of the stellar structure with the rate of G variation. The free-fall time (also known as the hydrostatic time-scale, $\tau_{ff} \sim 1/\sqrt{G\rho}$, with ρ the density of the object) gives the typical scale in which a dynamical stable star reacts to a slight perturbation of hydrostatic equilibrium. In the case of usual stars, this scale is extremely short: it is of order of minutes for a star like the Sun and of order of seconds for white dwarfs (WDs). This is negligible when compared with the rate of variation of G , which occurs over a scale similar to that of the age of the universe. Then, in most phases of the star evolution, one may safely consider them in hydrostatic equilibrium; and so would be even if G is a variable function.

WDs, in particular, do not generate gravitational fields too large as to necessarily consider them as relativistic objects. Retaining terms up to the post-Newtonian approximation, the first correction to the Newtonian equilibrium is proportional to $P/\rho c^2$, largely less than 1 for typical star pressures and densities. Then, we may adopt the methodological procedure of the right side of Fig. 1: take a simple theory and a complex object, and try to isolate observable effects. In this case, we are able to see the evolution of the *same* object through cosmic time, because under the Newtonian hydrostatic equilibrium approximation, evolution is just a sequence of static configurations.

From the above considerations, it is natural to expect that the direct introduction of a time

varying G into the equations that represent the equilibrium structure of WDs will be a safe procedure. The star will be able to see that variation, reacting to it immediately: it can not remember the value of G at its formation but it is forced to evolve changing with it. A question still remains: if the rate of change in G is slow enough as to agree with other gravity tests, are these changes appreciable?

Indeed, if G varies, WDs evolution could be sensitive enough as to provide a good independent method for measuring its change. There are two immediate reasons for this: firstly, they have lived during most of the life of the universe, and have time to integrate extremely small values of the rate of change of G . In addition, at their latest stages of evolution, their luminosity arise from a delicate balance between gravitational and thermal energies, and any change in G might strongly affect that equilibrium. This may produce a different luminosity function.⁴

We have run a detailed stellar evolution numerical code, with state-of-the-art physical inputs corresponding to WDs at each time of their evolution, and with a careful account of a variable G . Making no *a priori* assumptions on the thermal behavior of the interior of the WD, we have computed the evolution of C/O (hydrogen envelopes) models with masses ranging from $M = 0.4M_\odot$ to $M = 1.0M_\odot$, at intervals of $0.1M_\odot$, and with zero metallicity. We assumed the mass of the hydrogen and helium layers to be $M_H/M = 10^{-5}$ and $M_{He}/M = 0.01$, and a decreasing value of G . We took $G \propto t^{-2/4+3\omega}$, with ω a constant, which is the cosmological solution of the BD theory in the matter era. Details of the simulations, the code, and the input physics may be found in Ref. [11] and references cited therein.

The first striking result is that, because of the presence of a variable G in the energetic balance equations, the cooling of WDs is strongly accelerated, particularly at low luminosities. Owing to the fact that more massive WDs have smaller radii, this effect turns out to be far more dramatic in massive WDs. In Fig. 2 (left) we show the relationship between the logarithm of the luminosity of the WD and its age. The effect of $\dot{G} \neq 0$ is noticeable: while for very high values of ω the function is rather similar to that of the standard case, the time spent by a $0.6 M_\odot$ WD in reaching $\log L/L_\odot = -5$ fall to a fifth for $\omega = 1000$. Not surprisingly, for higher stellar masses, these

⁴The luminosity function (WDLF) is defined as the number of WDs with a given luminosity per cubic parsec and per unit luminosity.

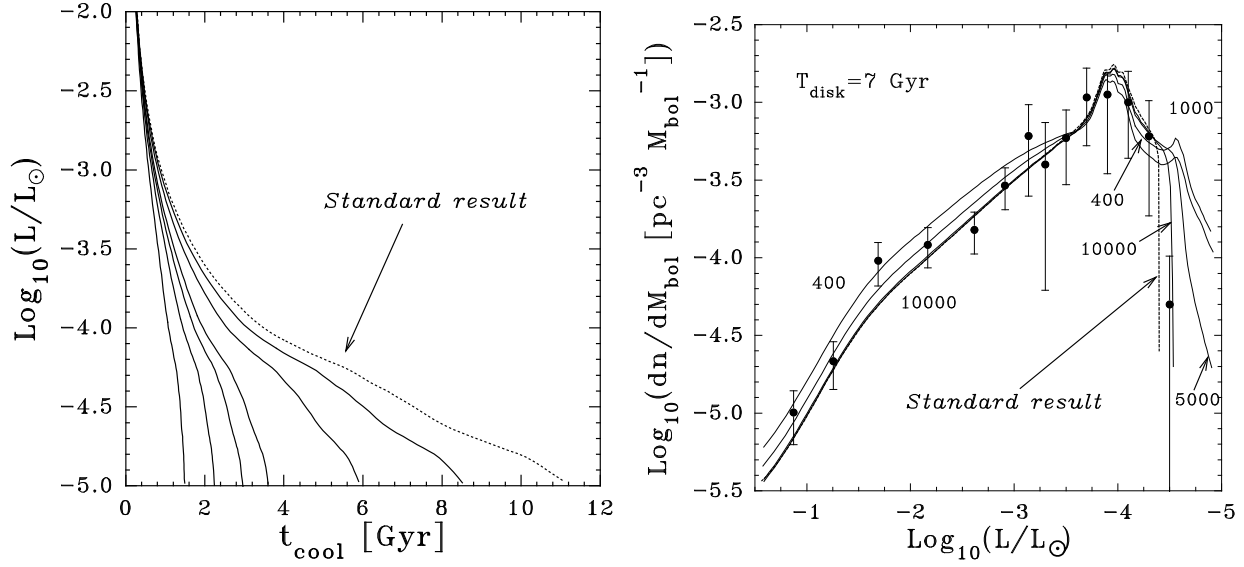


Figure 2: The figure on the left shows the acceleration of the cooling of a $0.6M_{\odot}$ WD assuming an age for the universe of 12.5 Gyr for different values of the ω parameter, from left to right, going from 400 to 5000 ($G \propto t^{-2/4+3\omega}$). The figure on the right shows the observed points of the luminosity function together with the theoretically computed ones. The dashed line corresponds to the G -constant solution and the solid ones to the G -varying case, with ω ranging from 400 to 10^3 . The galactic disk age was assumed to be 7 Gyr, possibly the lower plausible limit. Bolometric magnitudes are $M_{\text{bol}} = -2.5 \log(L/L_{\odot}) + 4.75$. Observed points are taken from Leggett et al., Ap. J. 497, 294 (1998). The theoretical computation of the luminosity function follows Iben and Laughlin, Ap. J. 341, 312 (1989).

differences become larger. In Fig. 2 (right) we show the theoretical WDLF along with the observed points. Amazingly, we find that even considering a value of ω as large as 10^3 , differences are found between the $\dot{G} \neq 0$ and $\dot{G} \equiv 0$ behaviors. Values of $\omega < 5000$ should be discarded, as we see no agreement between observed and computed WDLFs (recall the solar system limits $|\omega| > 500$). That would imply a current value of $|\dot{G}/G|$ of order 10^{-14} , which is between 1 and 3 orders of magnitude more restrictive than previous bounds.⁵ It is important to stress that the lowest luminosity point in the observed distribution is the least precisely determined, and it is still quite possible that its position may suffer variations in the near future.

On the basis of these results, we conclude that the evolution of WDs (even in the context of the assumed approximations) is a very powerful tool to probe the variation in the value of G with a greater degree of sensitivity than that provided by other experiments. When a comprehensive

⁵This limit is valid for our cosmic time because the WDLF is constructed with stars in our neighbourhood, all of them seeing our current value of G .

knowledge of the WDLF be available, a comparison with these results may help to decide whether or not a $\dot{G} \equiv 0$ theory is a better description of gravitation.

3 Final comments

We have intended to analyze two approximate schemes to an unique problem: the determination of the influence of a varying- G cosmology upon astrophysical structures. This would mean to exactly solve relativistic field equations, and simultaneously provide the behavior of the matter and the metric (together with the scalar related with G). The real situation would imply space and cosmic time dependences, and it is hardly tractable. However, as soon as the theory predicts a cosmological variation of G , the real evolution of the objects will also require a running G -value (unless memory effects are operative). By following the two methodological lines depicted in Fig. 1, we have studied these phenomena for boson and WD stars.

WD cooling was recently studied in Ref. [12], with the same energetic balance input and similar qualitative results. However, the crucial assumption of Ref. [12] was an isothermal interior, something that we explicitly found as not valid. This absence of isothermicity produces an even stronger acceleration of the cooling and more notorious observational effects in the luminosity function. Time variation of G was also analyzed as the possible cause of the discrepancy between the cosmic expansion age and the apparent globular cluster age [13]. Observational signatures of boson stars are currently being investigated [14], and the possibility of using these as discriminators of different theories of gravity is under analysis, especially in microlensing phenomena [15].

In static situations, the most dramatic effect recently discovered is spontaneous scalarization [16]. It happens in neutron stars models in general scalar-tensor theories, and provides nonperturbative effects which induce large deviations from GR behavior. The gravitational equilibrium configuration of a neutron star -at a given cosmic time- suffers spectacular changes, increasing their maximum masses. This effect has impact on neutron star binary coalescence and pulsar tests.

A comprehensive knowledge of all the influence that a varying- G cosmology would have on astrophysics seems to be far, especially when one considers the large class of astrophysical objects with long lifetimes, and not just isolated stars. It is certain, however, that studying this problem provides us with the possibility to gain a deep insight of fundamental physics, and that it is worth

exploring.

Acknowledgments

It is a pleasure to thank L. G. Althaus, O. G. Benvenuto, A. R. Liddle, F. E. Schunck, and A. W. Whinnett for the different collaborations which made possible the results commented here, and to L. Anchordoqui and G. E. Romero for a critical reading of this essay. The research on which this essay is based was supported by CONICET (Argentina), the British Council (UK), while the author was a Chevening Scholar, and Fundación Antorchas (Argentina) under grant A-13622/1-98.

References

- [1] P. A. M. Dirac, Proc. Roy. Soc. A165, 199 (1938).
- [2] C. Brans and R. H. Dicke, Phys. Rev. 24, 925 (1961).
- [3] C. M. Will, *Theory and Experiment in Gravitational Physics* (CUP, Cambridge, 1993); gr-qc/9811036.
- [4] M. J. Drinkwater et al., Mon. Not. R. Ast. Soc. 295, 457 (1998).
- [5] J. D. Barrow, Phys. Rev. D59, 043515 (1999).
- [6] J. D. Barrow, Phys. Rev. D46, 3227 (1992).
- [7] D. J. Kaup, Phys. Rev. 172, 1331 (1968); R. Ruffini and S. Bonazzola, Phys. Rev. 187, 1767 (1969).
- [8] M. Colpi, S. L. Shapiro and I. Wasserman, Phys. Rev. Lett. 57, 2485 (1986).
- [9] M. S. Madsen and A. R. Liddle, Phys. Lett. B251, 507 (1990); I. I. Tkachev, Phys. Lett. B261, 289 (1991); E. Seidel and W. Suen, Phys. Rev. Lett. 72, 2516 (1994).
- [10] D. F. Torres, Phys. Rev. D56, 3478 (1997); D. F. Torres, A. R. Liddle and F. E. Schunck, Phys. Rev. D57, 4821 (1998); D. F. Torres, F. E. Schunck and A. R. Liddle, Class. Quant. Grav. 15, 3701 (1998); A. W. Whinnett and D. F. Torres, “Charged boson stars: equilibrium, stability and evolution”, submitted to Phys. Rev. D gr-qc/9905017.

- [11] O. G. Benvenuto, L. G. Althaus and D. F. Torres, Mon. Not. R. Ast. Soc., to appear (1999).
- [12] E. García-Berro et al., Mon. Not. R. Ast. Soc. 277, 801 (1995).
- [13] S. Degli Innocenti et al., A & A 312, 345 (1996)
- [14] F. E. Schunck and A. Liddle, Phys. Lett. B404, 25 (1997).
- [15] A. W. Whinnett, private communication (1998).
- [16] T. Damour and G. Esposito-Farese, Phys. Rev. Lett. 70, 2220 (1993).